# Relating fishing mortality to trawl effort on the North-West shelf of Western Australia 

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### 1.0 NON-TECHNICAL SUMMARY

The aim of this project was to determine the level of fishing effort in the Pilbara Fish Trawl Fishery that would enable catches to be sustainable. This required the determination of the relationship between fishing effort and fishing mortality of five key species in the fishery.

The multispecies species nature of the fishery meant it was necessary to choose several key species as indicators of the response of the stocks to exploitation. Five species, which make up the majority of the catch were selected: two large species, red emperor and rankin cod, together with three small species, flagfish, lesser spangled emperor, and rosy threadfin bream.

The two large species are vulnerable to over-fishing because of they are long lived and have low natural mortality. The three small species are less vulnerable to overexploitation due to their high natural mortality and short lived nature. At the 1994 and 1995 levels of effort, the two large species, red emperor and rankin cod were overexploited. Increases in efficiency and the upgrading of the fleet to larger boats will put more pressure on the vulnerable species in the future, if the fishery were to expand unchecked within the current limits of number of boats and months access.

Four management options were put to industry, each involving different levels and distribution of effort, together with there expected impacts on the fish stocks.
Option 1. Allow effort to expand to full potential with uncontrolled distribution.
Option 2. Allow effort to expand to full potential and distribute effort across the fishery to reduce exploitation of the long lived species.
Option 3. Reduction of effort by $17 \%$ and distribute effort across the fishery to reduce exploitation of the long lived species.
Option 4. Reduction of effort by $25 \%$ and distribute effort across the fishery to reduce exploitation of the long lived species.

The expected result of Options 1 and 2 would be a serious depletion of the large high valued species. Option 3 would be expected to cause a decline in the large species to a smaller average size and tonnage. Option 4 would possibly secure the large species catch at existing levels. There would be reduced catch of some small species with possible under-exploitation. Lesser-spangled emperor, the major component of the catch, would have little reduction in catch due its concentration in the west. In this area, high effort can continue due to the low proportion of the larger long-lived species in the catch. If one 6-month licence was removed from the fishery
and there was a 17\% effort reduction, with redistribution of effort, the impact would be very similar to option 4.

The portion of the fishery in the deeper water, seaward of 100 m , is very lightly fished at present. The species in this area are expected to be highly vulnerable to overfishing. It has been recommended to industry that the access to this area be restricted until the nature and extent of this resource can be evaluated.

A management plan is to be implemented from the beginning of 1997 and will incorporate effort reduction and specified distribution of effort. A Vessel Monitoring System will commence at the same time to ensure that the fleet complies with the management arrangements.

### 2.0 BACKGROUND

The North-west Shelf of Western Australia has a long history of fishing by foreign vessels, the most significant being the operation of the Taiwanese pair-trawl fleet from 1971 to 1990. In 1987 some Western Australian prawn fishers working out of Point Samson began to supplement their prawn catch with trawled finfish. As the success of otter-trawling for finfish became apparent, the fishery developed rapidly from a catch of 12 tonnes in 1987 to 1784 tonnes in 1992. In 1992, the combined catch of trawl and trap fish on the North-West Shelf exceeded the most optimistic sustainable yield estimates. These yields estimates were based mainly on the series of annual Taiwanese logbook catch and effort data and the perception of the Pilbara trawl fleet was that there was a larger resource which was not being fully exploited.

CSIRO conducted extensive surveys in this area giving information on the distribution of several species within the fishery and also in shallower waterlandward of the fishery. This was valuable in giving an insight into the extent of the stocks in areas not available to the trawl fishers.

The impact of Australian fleet on this resource is poorly known because of the rapid development of the fishery and the consequent lack of long term catch and effort data. The paucity of information on the biology of numerous species in this fishery adds to the difficulty of management of this fishery. Consequently, discussions were started with industry on the benefits of an experimental approach in which rapid and reliable estimates of the level of sustainable fishing effort could be determined.

### 3.0 NEED

There was a need to determine the level of fishing effort by the Pilbara fish trawl fleet that would enable catches to be sustainable.

### 4.0 OBJECTIVES

The objective was to identify this level of effort by determining the relationship between fishing effort and fishing mortality of major species in the fishery.

In order to achieve this objective :

1. A small area of the fishery would be fished intensively by the trawl fleet for one year and the remainder of the fishery was closed to fishing.
2. Two extensive surveys would be conducted, across fished and unfished areas, at the beginning and the end of the time period, in order to determine the change in abundance of key species due to fishing. The purpose of this strategy was to ensure a measurable effect on the stocks and then by extrapolation, set effort levels for the full extent of the fishing zone.
3. Six species would be selected as indicators of the response of the stocks to exploitation. These would be species which made up the majority of the catch and would consist of some long lived species and some short lived species.
4. A sample of the survey catch would be aged, to determine the age structure of the key species at the beginning and the end of the experimental period.
5. Fishing mortalities would be determined from the change in the age structure at the effort level between the two surveys.
6. A yield-per-recruit model would interpret the magnitude of these values of fishing mortality using the limited data available on growth and natural mortality.
7. The expected impact on the fish stocks of different levels of effort would be presented to management and industry to assist in the formulation of a management plan for the fishery.

### 5.0 MATERIALS AND METHODS

### 5.1 Site description.

Three areas were selected, one open to fishing, and two closed to fishing for the duration of the experiment (Figure 1). The area open to fishing was adjacent to Point Samson, the port servicing this fishery. The location would minimise the need for boats to travel through the closed areas and hence reduce the difficulty of enforcing the closure. The gas pipeline from the North Rankin "A" platform to the Burrup Peninsula was chosen as the western boundary since trawlers cannot trawl over this obstruction for safety reasons and regular helicopter flights by the operators of the oil platform along the pipeline enabled the infringements of the closure to be monitored.


Figure 1. Experimental areas in the Pilbara Trawl Fishery and the grid used for recording catches.

The open area extended from the pipeline to 117030 ' . By restricting the area of the fishery to this size, it was anticipated that the stock would be subjected to sufficient effort to detect a difference between the changes in fish densities in the fished and unfished areas after one year. This restricted area was gazetted as the fishery on November 30, 1993 and the full area of the fishery was re-opened on March 3, 1995.

### 5.2 Survey design and sampling.

The experimental area was bounded by the meridians of longitude $115^{\circ} 30^{\prime} E$ and $118^{\circ} \mathrm{E}$ and the 50 m and 100 m depth contours. The two areas closed to fishing were from North Rankin "A" gas pipeline to the meridian 115030'E and also from 117030 'E to $118^{\circ} \mathrm{E}$ (Figure 1). The length of trawl shots by the commercial operators was about 8 nautical miles, and on this basis the experimental area was divided into sixty 10 ' by 10 grids

Two commercial vessels were used to conduct survey trawls in the fishery in both 1993 and 1994. The vessel owners retained the catch to defray the cost of the
survey. The nets used to conduct the survey were 18 fathom cut-away wing trawl nets manufactured by Eden Fishing Equipment, identical to those normally used by the fishing vessels. The nets on the two boats had a 232 inch ( 5890 mm ) and 220 inch ( 5630 mm ) fishing circle, 9 inch ( 230 mm ) mesh on wings, shoulders and bottom belly. The mesh on the body panels was 6 inch ( 150 mm ) and 4.5 inch ( 115 mm ). The extension and cod-end was double 4 inch ( 100 mm ) mesh. The net was spread with 1.2 m by 0.6 m otter boards using 90 m sweeps with 25 m bridles. Although the two vessels were using different sized nets, a new net of identical design was used on each survey.

Between October 16 and December 20 1993, 352 shots were conducted in the experimental area. The second survey conducted in 1994 completed 392 shots between October 12 and December 12. The latitude, longitude, and time were recorded at the start and finish of each shot, as well as average speed and average depth. There were eight trawl shots in most grids but in areas of rough ground there was a reduced number of shots due to the risk of net damage. In areas of low catch rates (often on the edge of the fishery) there were fewer trawls since the foregone catches by the operators would have been high.

On one vessel the same skipper conducted the survey in each year, and consequently the shots were repeated (with a few exceptions) using the same trawl direction and speed. It was difficult in some cases to follow the track of the shots of the previous survey due to strong adverse currents. On the other vessel a different skipper was used each year and the repetition was not as close, with many shots not being conducted in the same order each year and shots being conducted in the opposite direction to that in the previous survey.

### 5.3 Target species

Of the many species caught in this fishery, five of the major species in the catch were used in this study. The target species selected were two large species, red emperor (Lutjanus sebae) and rankin cod (Epinephelus multinotatus), and three small species, flagfish (Lutjanus vitta), lesser spangled emperor (Lethrinus sp. undescribed, previously referred to as L. choerorynchus [Sainsbury et. al. 1985]), and rosy threadfin bream (Nemipterus furcosus). A sixth species, Lutjanus
erythropterus was initially included but later dropped due to the very erratic catches of this species.

### 5.4 Length Frequencies

During the two surveys the fork length (length from the snout to the caudal fork) of the five key species was measured to the nearest centimetre and recorded with the time and location of collection. All of the two large species, red emperor and rankin cod, were measured. If the catch of the smaller species (flagfish, lesser spangled emperor, and rosy threadfin bream) was less than about 25 kg then all fish were measured. For the larger catches, a sample of about 25 kg of each species was selected at random and measured. The frequencies of each length class were weighted by the proportion of the catch (by volume) measured.

### 5.5 Age determination

From the sample used for length determination, a sub-sample was selected at random for age determination, other than for rankin cod where all were selected. The fork length (mm) was measured before sagittal otoliths were removed, cleaned, and dried on paper towel and then stored in paper envelopes with collection number, location, date, and fork length. The otoliths of rankin cod and flagfish were fragile and only one otolith was collected in some cases. The number of fish measured and the number of fish from which otoliths were collected is shown in Table 1.

Table 1. Number of fish measured and otoliths collected during research surveys.

|  | Number of fish measured |  | Otoliths collected |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 1993 | 1994 | 1993 | 1994 |
| flagfish | 16700 | 18000 | 996 | 1064 |
| lesser spangled emperor | 15100 | 16600 | 1005 | 1014 |
| threadfin bream | 15600 | 14900 | 990 | 997 |
| red emperor | 2300 | 3400 | 1000 | 1140 |
| rankin cod | 520 | 650 | 510 | 602 |

Figure 2. Teflon mould used for embedding otoliths in epoxy resin.

Figure 3. Gemmasta saw used to section otoliths.
The otoliths were mirror images and where possible the right otolith of each pair was selected for ageing. Rows of three to six otoliths (depending on their size) were embedded in epoxy resin using teflon moulds (Figure 2). Epoxy resin was used as there was no shrinkage or distortion, as occurred with casting resin. The rows of otoliths were sectioned transversely to a thickness of 0.4 mm using a Gemmaster high speed saw (Figure 3) with a 100 mm by 0.1 mm diamond tipped saw blade.

Generally three sections were cut from each row to ensure that a section containing the central core was obtained for each otolith. The otoliths containing the core were set on 76 mm by 50 mm glass slides with casting resin and covered with cover slips. Each slide contained between 14 and 24 sectioned otoliths, depending on their size.


Figure 4. Section of red emperor otolith showing opaque (white bands) which were counted for allocation of ages.

Otoliths were viewed under a compound microscope using reflected light and also transmitted light. Under reflected light the sectioned otoliths showed a series of concentric opaque rings which appeared white, separated by translucent rings which appeared dark (Figure 4). The number of opaque rings on all otoliths were read by two researchers. If the count of the opaque zones differed by more than one, the otolith was re-read by the author and the author's count for the opaque zones was used. To address the difficulty of determining the first ring, fish less than one year old were obtained from Pilbara prawn trawl by-catch, sectioned and read. Viewing of these sections helped determine the position of the first ring in older fish. Opaque zones on otoliths were assumed to be annual on the basis of the studies (McPherson and Squire 1992, Ferreira and Russ 1992, Moran 1993) confirming annual periodicity of otolith rings in tropical Lethrinids, Lutjanids, and Serranids. While recognising that validation for the species studied in this fishery is still required, the number of opaque rings was used for age estimation.

### 5.6 Input data for parameter estimation.

## Age-length key

The aged fish were grouped into 10 mm length intervals and this data was converted into a table of frequencies of age-at-length (an age-length key, Ricker 1975). It was anticipated that the effort contrast between the "open" area and that "closed" to fishing would result in a different age structure in the different areas, and so a different age-length-key was used for each of the three fishing areas.

## Catch per grid

The length-frequency data in each trawl grid from the trawl surveys was converted to age-frequency data by assigning ages to fish according to their length, in the same proportion as the ages in the age-length key.

To account for variation in trawl distances between shots, the number of fish caught in each age class for each trawl shot was scaled by the proportion of the trawl distance of that shot to the average distance over the two surveys ( 7.112 n miles) to give a scaled catch, $\mathrm{x}_{\mathrm{t}, \mathrm{i}}$, for fish of age t , in grid i for the 1993 survey, and $\mathrm{x}_{\mathrm{t}+1, \mathrm{i}}$ for fish one year older in the 1994 survey.

The arithmetic means of the scaled catch for each shot within the grid in the 1993 and 1994 survey ( $\bar{x}_{t, i}$ and $\bar{x}_{t+1, i}$ respectively) were used as the measure of abundance for each age class $t$ in each grid $i$.

## Effort in the trawl grids.

Hours of effort ( $E_{i} \geq 0$ ) by the commercial fleet in each trawl grid between November 15, 1993 and November 16, 1994 was determined from the time and location of trawl shot recorded in fishers logbook data.

### 5.7 Mortality Model

A model was used to predict the 1994 scaled mean catches of each age class, $\hat{\bar{x}}_{\mathrm{t}+1, \mathrm{i}}$, from the 1993 scaled mean catches, $\overline{\mathrm{X}}_{\mathrm{t}, \mathrm{i}}$ by estimation of parameters measuring mortality and diffusion.

The change in the mean scaled catches between the two years can be attributed to the two major components, natural mortality and fishing mortality (effort $\times$ catchability). The contrast in effort between trawl grids enabled the catchability of each age class, $\mathrm{q}_{\mathrm{t}}$, to be estimated. Natural mortality and catchability are assumed to be constant over all trawl grids.

Despite attempting to keep trawling conditions the same for the two surveys, there was an increase in efficiency between the two surveys, on both vessels, resulting in increased catches in the second survey. Natural mortality was confounded with this efficiency increase precluding the estimation of the true natural mortality. The confounded natural mortality is denoted in the model by $\mathrm{M}^{*}$ to emphasise its difference from $M$, the true natural mortality.

A diffusion parameter ( $0 \leq \mathrm{D} \leq 1$ ), considered constant over all trawl blocks, allowed for localised migration (diffusion) in both directions between adjacent grids giving a net result of movement from areas of high fish density to low fish density. It is assumed that diffusion is localised and does not occur over a greater distance than an adjacent trawl grid. The fish were considered to diffuse immediately after the first survey and be subject to mortality one year later.

A model was used to determine values of the parameters $\mathrm{M}^{*} \geq 0$ (natural mortality confounded by efficiency of survey fishing), $q \geq 0$ (catchability), and diffusibility ( $0 \leq \mathrm{D} \leq 1$ ). The parameters are determined separately for each age class.

The predicted catch for the following year for each age class is:

$$
\begin{equation*}
\hat{\bar{x}}_{t+1, i}=\left\{\bar{x}_{t, i}(1-D)+\frac{D}{4} \sum_{j=1}^{4} \bar{x}_{t, j}\right\} \cdot \exp \left(-M^{*}-q_{t} E_{i}\right) \tag{1}
\end{equation*}
$$

where the meaning of the variables is
$\bar{x}_{t, i} \quad$ mean scaled catch in trawl grid $i$, at survey time $t$ years,
$\hat{\bar{x}}_{t+1, i}$ predicted scaled catch for trawl grid $i$ at survey time $t+1$,
$\bar{x}_{t, j} \quad$ mean number of fish in adjacent trawl grid $j$ at time $t$ years
M* natural mortality (confounded with efficiency)
q catchability in 1994
$\mathrm{E}_{\mathrm{i}} \quad$ effort in hours trawled in grid i
D diffusivity parameter

Estimates of the parameters $\mathrm{M}^{*}, \mathrm{q}$, and D were obtained by minimisation of the objective function $\operatorname{SS}(\hat{\theta})$ shown as equation (2) using the Levenberg-Marquardt method (Dennis and Schnabel,1983)

$$
\begin{equation*}
S S(\hat{\theta})=\sum_{i=1}^{n} \frac{N_{t+1, i}}{S_{t+1, i}^{2}}\left(\ln \left(\bar{x}_{t+1, i}+1\right)-\ln \left(\hat{\bar{x}}_{t+1, i}+1\right)\right)^{2} \tag{2}
\end{equation*}
$$

where n is the number of grids in which fishing occurred during the survey and $\hat{\theta}$ is
the parameter value vector

$$
\hat{\theta}=\left(\begin{array}{c}
M^{*} \\
q \\
D
\end{array}\right)
$$

The error term, or the residual in the objective function is given by

$$
\begin{equation*}
\varepsilon_{\mathrm{i}}=\ln \left(\overline{\mathrm{x}}_{\mathrm{t}+1, \mathrm{i}}\right)-\ln \left(\hat{\bar{x}}_{\mathrm{t}+1, \mathrm{i}}\right) \tag{3}
\end{equation*}
$$

The objective function was weighted by the number of shots per grid $N_{t+1, i}$ in grid $i$ at time $t+1$. The objective function was also weighted by the inverse of the observed
variance for each grid $\left(S^{2}{ }_{t+1, i}\right)$ to take account of the variability in the catch rates within each grid.

### 5.8 Statistical assumptions

The key assumptions made for the analysis (Bates and Watts 1988) are

1. that the model is correctly specified with all relevant predictor variables included in the regression equation.
2. the observed catch and effort are measured without error.
3. the error term for parameter estimation, $\varepsilon_{i}=$ observed catch - predicted catch is an independent normally distributed variate with a mean of zero and constant variance.

To satisfy the third assumption, a natural logarithm transformation of the mean scaled catch $\bar{x}_{t+1, i}$ and the predicted mean scaled catch $\hat{\bar{x}}_{t+1, i}$ was used to normalise the error term. This transformation can also be used to produce constant variance in the error term, where a relationship exists between the mean and the variance, thus satisfying the assumption of constant variance. of the error term. The logarithm transformation also ensured that the objective function placed similar emphasis on fitting blocks with both high and low fish counts.

It was necessary to add a constant to $\bar{x}_{t+1, i}$ since there were zero catches in some grids. The constant 1 was added to $\bar{X}_{t+1, i}$ since this is the smallest number of fish $>0$ caught in any one trawl shot (Zar, 1984 p 238). The variance within each grid was calculated from the logarithmic transformed scaled 1994 data, $\ln \left(\bar{x}_{t+1, i}+1\right)$. The variance of the 1994 logarithmic transformed data was used since the objective function is applied to transformed data.

A small value for the variance was observed in some grids where the catch and number of trawls was small, giving a high weighting to these grids. When this occurred, the minimum variance was taken as the smallest variance for those grids in which the full 8 trawl shots were taken. Grids 4820 and 4527 (Figure 1) were removed from the analysis as only one shot was taken in these grids.

### 5.9 Computation of standard errors

Standard errors of the parameters $\mathrm{q}, \mathrm{M}^{*}$, and D were calculated to determine the precision of parameter estimates. Determination of exact confidence intervals for nonlinear least squares problems is computationally difficult and generally approximation techniques are used. A linearization method, in which the nonlinear function is approximated by a linear function at the solution (Donaldson and Schnabel, 1987) was used in this analysis.

There are a number of linearization methods for constructing approximate confidence intervals, all of which assume that the solution surface is planar throughout the area covered by the confidence intervals (Donaldson and Schnabel 1987). Of these methods, Donaldson and Schnabel (1987) recommend using the Jacobian matrix to estimate the covariance matrix as it is simpler to use, less time consuming to compute, and more numerically stable than other methods.

In this study the parameter covariance matrix, $\hat{\mathbf{V}}$, was estimated from the Jacobian matrix evaluated at the value of the three fitted parameters $D, M^{*}$ and q. From the covariance matrix, linear approximations of the $100(1-\alpha) \%$ confidence limits for the three parameter estimates $\theta_{j}(\mathrm{j}=1, \ldots 3)$ were found from

$$
\begin{equation*}
\theta_{\mathrm{j}} \pm \hat{\mathbf{V}}_{\mathrm{jj}}^{1 / 2} \mathrm{t}_{\mathrm{n}-\mathrm{p}, \alpha / 2} \tag{4}
\end{equation*}
$$

where $t_{n-p, \alpha / 2}$ denotes the upper $\alpha / 2$ precentile of the $t$ distribution with $n-3$ degrees of freedom (Donaldson and Schnabel 1987).

A correlation matrix between the parameters was determined from the covariance matrix $\hat{\mathbf{V}}$ (Johnson and Wichern, 1992) in order to investigate the adequacy of the model structure.

The coefficient of determination $\left(\mathrm{R}^{2}\right)$ (Johnson and Wichern, 1992) was used to determine the adequacy of the model to estimate parameter values for each age class of a species.

### 5.10 Evaluation of adequacy of the model.

A plot of predicted values versus actual values for abundance in 1994 was used to check the adequacy of the model to fit the data. The correlation between the parameters, shown in the correlation matrix, was used as a crude check for redundant parameters in the expectation function.

A plot of residuals versus predicted values was examined to detect outlying observations and to examine the adequacy of the assumption of constant residual variance. A trend in the residuals in this plot would reveal a possible structural deficiency of the model. A quantile-quantile plot (Johnson and Wichern, 1992) was used to check the assumption of normality of the model error term. A spatial plot of the residuals was used to check whether the residual values were independent of grid location.

### 5.11 Average effort and fishing mortality in each trawl grid.

In order to determine the degree of exploitation of each species, it is necessary to make a comparison between the fishing mortality which, on average, the fish experience and a biological reference point obtained from a yield-per recruit model. To determine the fishing mortality of each age class for each species, it is necessary to calculate the average effort that the fish have been subjected to, taking into account the varied distribution of fish numbers and fishing effort. This was achieved by weighting the effort in each trawl grid by the 1993 survey catch rate for each species. The 1993 catch rate was used as this represents the densities before intensive fishing occurred in 1994. The fishing mortality was calculated as the product of the catchability (calculated from the model) and the weighted average effort for each species. Details of the method used to calculate the average weighted effort and fishing mortalities, and their variances are shown in Appendix 2.

The fishing mortality could be estimated for only a restricted number of age classes in the middle of the age range due to small numbers fish at the extremes of the age range. To obtain estimates of the fishing mortality over the whole age range, a function was fitted simultaneously to the fishing mortality estimates arising from the model (equation 1) and the age-frequency distribution of the sample selected for ageing from the 1993 and 1994 survey data.

The fishing mortalities of different age classes are considered to be related to the pattern of recruitment and net selectivity. The fishing mortalities, F, were assumed to be described by a logistic function of age, the function was used by Sparre and Venema (1992) to represent the selection ogive of a trawl. The details of the method of estimation of the parameters of the logistic function are shown in Appendix 2.

### 5.12 Yield per Recruit model.

To assess the level of exploitation of each species during the period of the experiment, the fishing mortality for each species was compared to the biological reference point $F_{0.1}$ This is recognised as a conservative measure of appropriate fishing mortality (Deriso 1987, Mace and Sissenwine 1993). A yield per recruit (YPR) model was used to determine values of this biological reference point for the five target species.

The YPR is the ratio of the weight of fish removed from a cohort due to fishing to the number of fish which recruit to the fishery. The YPR analysis assumes that growth rates and mortality are not density dependent. The effects on yield of declining recruitment due to reduced egg production are not taken into account in a YPR analysis.

The Ricker YPR method (Ricker 1954) incorporating the conventional Baranov catch equation (Baranov 1918, cited in Ricker 1975) was used to calculate yield-per-recruit. The parameter information required for each species for the YPR model is the length-at-age, the length-weight relationship and estimates of the instantaneous rate of natural mortality, M.

The average length for each age class was obtained from the aged fish from the 1993 survey. The length-weight relationship was obtained from linear regression of $\ln$ (length) and $\ln$ (weight), where fish lengths and weights were obtained from samples of fish selected at random from fish caught by the trawl fleet in 1994. The values of the instantaneous rate of natural mortality ( $M$ ) were derived from the longevity of the species (Hoenig 1983). For each species, a range of alternative values of $M$ was considered to investigate the sensitivity of the YPR model to changes in M .

The yield per recruit model calculates the yield as a function of fishing mortality. The fishing mortality is often considered constant for all age classes above the age of recruitment, which is not the case in this study. The different fishing mortalities can be incorporated into the YPR model as the catchability, q, was assumed for different levels of effort. Each value of the independent variable, fishing mortality, in the YPR model, was weighted for each age class by the maximum fishing mortality, $\mathrm{F}_{\text {max }}$, the asymptotic fishing mortality of the fitted logistic function. That is, the values of fishing mortality for which yield per recruit was calculated, are expressed in terms of the fishing mortality for the most vulnerable age classes.

To determine $\mathrm{F}_{0.1}$ for each species, the slope of the YPR function is calculated at the origin. The value of fishing mortality at which the slope is one-tenth of the slope of the curve at the origin is defined as $F_{0.1}$. As the yields are expressed in terms of the most vulnerable age class, so too is the value determined for $F_{0.1}$.

A comparison of $F_{0.1}$ with $F_{\max }$ indicates the degree of exploitation. If $F_{\max }$ for $a$ species was greater than $F_{0.1}$ then the species was classified as over-exploited.

## Determination of degree of exploitation after increased effort.

In 1995, the fishery was re-opened and the fishing fleet was permitted to fish from $116^{\circ} \mathrm{E}$ to $120^{\circ} \mathrm{E}$ and seaward of the 50 m isobath. As catchability was assumed constant for different values of effort, the fishing mortality $\left(F_{k, t}\right)$ was re-calculated using the 1995 fishing effort in the survey area. The maximum value of fishing mortality $F_{\max }$ for each species was compared to $F_{0.1}$ to determine the degree of exploitation of each species for the different level and distribution of effort in 1995.

### 6.0 RESULTS

### 6.1 Parameter estimates

The parameter estimates, diagnostic statistics and plots, and confidence limits were determined using "Mathcad 6" (Mathsoft, 1995). The parameters $\mathrm{q}, \mathrm{M}^{*}$, and D estimated from the model are shown in Table 2. For each species, the age classes shown are those in which the model converged to give parameter estimates. In the other age classes the small numbers of fish precluded the estimation of parameters from the model. The values of $R^{2}$ for a particular species shows the comparison of the linear fit of the model for the different age classes.

Table 2. Estimates of catchability, q ; natural mortality, $\mathrm{M}^{*}$; diffusibility, D ; coefficient of determination, $\mathrm{R}^{2}$; fishing mortality from the model, F (with $90 \%$ confidence limits).


| 6 | 22 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} 2.82 \pm \\ 0.36 \\ \hline \end{gathered}$ | $0.51 \pm 0.02$ | $\begin{gathered} 0.02 \pm \\ 0.02 \end{gathered}$ | 0.616 | $0.093 \pm 0.013$ |
| red emperor |  |  |  |  |  |
| 8 | $\begin{gathered} 4.64 \pm \\ 0.46 \end{gathered}$ | $-0.58 \pm 0.02$ | $\begin{gathered} 0.74 \pm \\ 0.03 \end{gathered}$ | 0.323 | $0.136 \pm 0.017$ |
| 9 | $\begin{gathered} 3.97 \pm \\ 0.55 \end{gathered}$ | $-0.35 \pm 0.02$ | $\begin{gathered} 0.71 \pm \\ 0.03 \end{gathered}$ | 0.341 | $0.113 \pm 0.015$ |
| 10 | $\begin{gathered} 4.78 \pm \\ 0.54 \end{gathered}$ | $-0.59 \pm 0.03$ | $\begin{gathered} 0.73 \pm \\ 0.03 \end{gathered}$ | 0.391 | $0.140 \pm 0.017$ |
| 11 | $\begin{gathered} 4.72 \pm \\ 0.56 \end{gathered}$ | $-0.49 \pm 0.02$ | $\begin{gathered} 0.44 \pm \\ 0.02 \end{gathered}$ | 0.320 | $0.138 \pm 0.017$ |
| 12 | $\begin{gathered} 5.45 \pm \\ 0.67 \end{gathered}$ | $-0.36 \pm 0.02$ | $\begin{gathered} 0.58 \pm \\ 0.05 \end{gathered}$ | 0.241 | $0.160 \pm 0.020$ |
| rankin cod |  |  |  |  |  |
| 4 | $\begin{gathered} 9.41 \pm \\ 1.10 \end{gathered}$ | $-0.33 \pm 0.04$ | $\begin{gathered} 0.53 \pm \\ 0.04 \end{gathered}$ | 0.299 | $0.255 \pm 0.032$ |
| 5 | $\begin{gathered} 9.33 \pm \\ 0.80 \end{gathered}$ | $-0.27 \pm 0.04$ | $\begin{gathered} 0.73 \pm \\ 0.04 \end{gathered}$ | 0.264 | $0.253 \pm 0.032$ |
| 6 | $\begin{gathered} 6.45 \pm \\ 0.80 \end{gathered}$ | $0.12 \pm 0.04$ | $\begin{gathered} 0.56 \pm \\ 0.04 \end{gathered}$ | 0.282 | $0.175 \pm 0.029$ |
| 7 | $\begin{gathered} 7.14 \pm \\ 0.09 \end{gathered}$ | $0.31 \pm 0.03$ | $\begin{gathered} 0.98 \pm \\ 0.04 \end{gathered}$ | 0.058 | $0.194 \pm 0.025$ |
| 8 | $\begin{gathered} 5.78 \pm \\ 1.06 \\ \hline \end{gathered}$ | $0.30 \pm 0.03$ | $\begin{gathered} 0.48 \pm \\ 0.05 \\ \hline \end{gathered}$ | 0.498 | $0.157 \pm 0.030$ |

### 6.2 Adequacy of the model.

The correlation matrix for $\mathrm{q}, \mathrm{m}$, and D indicated that for all species the highest correlation was a negative correlation between $q$ and $m$, but in no case did it indicate a problem with the model. The correlation between the other pairs of variables was low.

A plot of the observed and predicted number of fish was investigated for each age class of each species. There was a good linear fit for nearly all age classes indicated by the values of $R^{2}$ in Table 2. For rankin cod the linear fit was acceptable for four age classes but was very poor for the 7 year old fish. There were several outliers for each of the five species and in several cases the predicted values differed by a factor of two from the observed value.

The plot of residuals versus predicted values indicated there was no departure from constant residual variance for the four species other than rankin cod. The residuals for these four are assumed to be homoscedastic. For rankin cod, the plot showed some departure from constant residual variance. High residuals were associated with low predicted values and low residuals associated with high predicted values
indicating there were some problems with the model performance for some grids for this species.

The quantile plots were examined to assess the assumption of normality of the residuals. For flagfish and rosy threadfin bream the plots were nearly linear. For lesser spangled emperor and rankin cod were two outliers and the linear trend was not as strong as the other two species. These outliers were examined and found to be due to zero catches in a grid in one survey and significant catches the same grid in the other survey. For these four species, the correlation was sufficiently high at the $1 \%$ significance level, that the assumption that the residuals are normally distributed cannot be rejected. For red emperor, there were three outliers on the quantilequantile plot, again due to zero catches in one survey. For this species, at the $1 \%$ significance level the residuals are not normally distributed. No erroneous data was detected and all outliers were retained for the analysis.

The spatial plot of the residuals for flagfish, threadfin bream and rankin cod showed no pattern to the distribution of the residuals, indicating that the residuals were independent of grid location.

For lesser spangled emperor there is a spatial pattern to the residuals, high values of the residuals occurring in the east of the survey area and low values occurring in the west. This indicates that the parameter estimates resulted in an over-estimation of the predicted values in the west of the fishery and under-estimated of the predicted values in the eastern grids. For this species the residuals are not independent of grid location.

The spatial plot of the residuals for red emperor, indicated several high residuals values in the east of the experimental area (grids $4633,4732,4734,4832$ ) which indicated that the model was under-predicting in these areas

### 6.3 Fishing mortality for the key species

The fishing mortality is calculated from the catchability q, assumed constant over all trawl grids, and the average effort in the survey area. The mean effort $\bar{E}$ (hours) and the variance $V(\overline{\mathrm{E}})$ for each species is shown in Table 3 and data used for the
calculation (1993 survey catch rates and the effort, $\mathrm{E}_{\mathrm{i}}$, in each survey grid) is shown in Appendix 2.

Table 3. Weighted mean effort $\bar{E}$ (hours) and the variance of $\bar{E}$.

| Species | $\overline{\mathrm{E}}$ | variance of $\overline{\mathrm{E}}$ |
| :---: | :---: | :---: |
| flagfish | 275.7 | 2181.3 |
| lesser spangled emperor | 233.6 | 2273.0 |
| threadfin bream | 326.0 | 2788.8 |
| red emperor | 293.4 | 2146.4 |
| rankin cod | 270.9 | 1982.6 |

The fishing mortality, F, and the $90 \%$ confidence limits were calculated for the age classes where q was known, and are shown in Table 2. For small and large ages, no estimate of $F$ was possible because of small fish numbers. The parameters of the logistic function fitted to the values of $F$ and those estimated to give rise to the survey age-frequency distributions are shown in Table 4. The value of "a", the asymptote of the logistic function, is taken as the maximum value of fishing mortality, $\mathrm{F}_{\text {max }}$, used to indicate the degree of exploitation of the key species.

Table 4. The parameters of the logistic function fitted to fishing mortalities.

| Species | a | b | c |
| :---: | :---: | :---: | :---: |
| flagfish | 0.083 | 5119 | 4.183 |
| lesser spangled emperor | 0.165 | 699490 | 4.338 |
| threadfin bream | 0.091 | 110.0 | 1.702 |
| red emperor | 0.142 | 124.0 | 0.856 |
| rankin cod | 0.252 | 2444 | 1.883 |

## Fishing mortality for flagfish

The fishing mortality was estimated by the model for fish with 1993 ages of 2 to 8 years, as illustrated in Figure 5 with $90 \%$ confidence intervals. The fitted logistic function is shown on the same figure.

The low value of fishing mortality for the age 7 fish (with a low value of $\mathrm{R}^{2}$ ) was not due to a small sample size for fish of this age.


Figure 5. Fishing mortality of flagfish with $90 \%$ confidence limits estimated by the model together with the fitted logistic function.

## Fishing mortality for lesser spangled emperor

The model estimates of fishing mortality, with $90 \%$ confidence intervals, and the fitted logistic function are shown in Figure 6. The low value of $F$ for age 5 fish was not associated with a low sample size of aged fish nor with a low value of $R^{2}$.


Figure 6. Fishing mortality of lesser spangled emperor with $90 \%$ confidence limits estimated by the model together with the fitted logistic function.

Fishing mortality for rosy threadfin bream
The expected fishing mortality with $90 \%$ confidence intervals (Figure 7) shows that the fishing mortality showed considerable variation, the non-overlapping confidence limits indicating they are significantly different at the $90 \%$ significance level.


Figure 7. Fishing mortality of rosy threadfin bream with $90 \%$ confidence limits estimated by the model together with the fitted logistic function.

## Fishing mortality for red emperor

The expected fishing mortality from the model with $90 \%$ confidence intervals (Figure 8) shows that the fishing mortality showed little variation over the age classes, showing no significant difference at the $90 \%$ significance level.


Figure 8. Fishing mortality of red emperor with $90 \%$ confidence limits estimated by the model together with the fitted logistic function.

## Fishing mortality for rankin cod

The expected fishing mortality from the model with $90 \%$ confidence intervals (Figure 9) indicates that the fishing mortality showed considerable variation over the age classes, but only the age 6 value of $F$ was smaller than the other values at the $90 \%$ significance level.


Figure 9. Fishing mortality of rankin cod with $90 \%$ confidence limits estimated by the model and the fitted logistic function.

### 6.4 Yield-per-recruit model

The parameters used in the Yield-per-recruit model consist of the constants of the length-weight relationship (Table 5), estimates of natural mortality derived from estimated maximum ages (Table 5), and the mean fish length for each age class in each fishing area (from the 1993 survey age-frequency distribution).

Table 5. Constants for weight-length relationship weight $=\mathbf{d} \times$ length $^{\mathbf{e}}$ (length in cm and weight in kg ), estimated maximum age and the corresponding natural mortality, M.

|  | d <br> $\times 10^{-5}$ | e | Maximum <br> Age | M |
| :---: | :---: | :---: | :---: | :---: |
| flagfish | 1.247 | 3.051 | $13,12,11$ | $0.32,0.35,0.38$ |
| lesser spangled emperor | 1.312 | 3.083 | $13,12,11$ | $0.32,0.35,0.38$ |
| threadfin bream | 5.392 | 2.825 | $13,12,11$ | $0.32,0.35,0.38$ |
| red emperor | 1.406 | 3.072 | $40,35,30$ | $0.10,0.12,0.14$ |
| rankin cod | 0.941 | 3.086 | $24,21,18$ | $0.17,0.20,0.23$ |

The asymptote of the logistic function was taken as the maximum value of fishing mortality, $F_{\text {max }}$, and the proportions of fishing mortality in each age class were derived using this value. Values of $\mathrm{F}_{0.1}$ were obtained for each species from the YPR model for each of the three values of natural mortality in Table 5 in each of the three
fishing areas. The minimum, median, and maximum of these values of $F_{0.1}$ are shown in Table 6.

### 6.5 Comparison of values of fishing mortality and $\mathrm{F}_{0.1}$

At the level of effort during the experimental period (18596 hours), the maximum value of fishing mortality, $F_{\text {max }}$, was greater than the median value of $\mathrm{F}_{0.1}$ for red emperor and rankin cod (Table 6), indicating that these two species are overexploited at this effort level. For the small species (flagfish, lesser spangled emperor, and rosy threadfin bream), $\mathrm{F}_{\max }$ was less than F0.1.

In 1995, the effort increased to 21709 hours, spread over a wider area due to the reopening of the fishery. The fishing mortalities, weighted on the 1993 survey catch rates, were calculated for the portion of the 1995 effort in the survey area (Table 6). Comparison with the values of $\mathrm{F}_{0.1}$ in Table 5 indicates that red emperor and rankin cod are over-exploited at this level of effort.
Table 6. The value of $F_{\max }$ for 1994 and 1995 effort levels and the minimum, median, and maximum values of the biological reference point $\mathrm{F}_{0.1}$.

|  | $\mathrm{F}_{0.1}$ | $\mathrm{~F}_{\max , 1994}$ | $\mathrm{~F}_{\max 1995}$ |
| :---: | :---: | :---: | :---: |
| flagfish | $0.46,0.53,0.60$ | 0.083 | 0.101 |
| lesser spangled emperor | $0.45,0.50,0.55$ | 0.165 | 0.228 |
| threadfin bream | $0.52,0.60,0.70$ | 0.091 | 0.093 |
| red emperor | $0.10,0.12,0.15$ | 0.142 | 0.162 |
| rankin cod | $0.20,0.24,0.28$ | 0.252 | 0.276 |

### 7.0 DISCUSSION

The parameter estimates and residuals determined by the mortality model revealed many areas for refinement and further investigation. Some areas of interest were the very low values of catchability for a few age classes, the spatial pattern of the residuals for lesser spangled emperor and red emperor, and the poor prediction in some grids due to high variation in catches between the two surveys for some grids.

### 7.1 Allocation of ages

The aberrant values of $q$ for some age classes (for example age 5 for lesser spangled emperor and age 7 for flagfish) may be due to very weak recruitment in previous years or may well be due to biased sampling, or incorrect allocation of ages. There is no indication of a low recruitment in the survey length-frequency distribution in 1993 or 1994. The sample of fish selected from the catch for ageing may have
been biased toward selection of large fish because they were not selected mathematically. It is unlikely that the method of selecting fish would cause large misrepresentation of one age class. If constant recruitment is assumed, the wrong allocation of ages can be investigated by allocating ages based on otolith weight. The otoliths weights are known for most of the survey otoliths and ages can be reallocated by determination of a relationship between otolith weight and age. The model can be used to determine new parameter estimates of ages determined from otolith weight.

### 7.2 Simultaneous calculation of catchability.

The model determined estimates of parameters $q, M^{*}$, and $D$ independently for each age class. The model estimates showed large variations in the parameter estimates between age classes. For flagfish and lesser spangled emperor, the smallest value of $q$ was less than half the largest value.

The approach in the present analysis was to use these values of $q$ to determine fishing mortalities in each age class and then fit a logistic function. Simultaneous estimation of $q$ for all the age classes was not a practical possibility in "Mathcad 6". An alternative approach could be to assume a function relating $q$ and age and then use all the survey data to estimate the parameters of the function. The catchability is not expected to be constant, as it is expected to be smaller for young fish as they escape through the net mesh and also because they are not fully recruited to the fishing grounds. On the other hand, $M$ is expected to be larger for the small fish than the adults (Sparre and Venema, 1992). It may be possible to develop a relationship between $q$ and age for the larger age classes where the natural mortality could be assumed to be constant. A starting point could be to assume the relationship between $q$ and age to be a logistic function. Assuming the increased efficiency in the second survey is constant for each age class and diffusibility is constant, it may be possible estimate the parameters of the logistic function, natural mortality (confounded with efficiency), and diffusibility, simultaneously for all age classes.

### 7.3 Small sample used for ageing

The fluctuations in the parameter estimates and the poor residual plot for lesser spangled emperor, could be due to the sample used for ageing being too small. The number of fish used for ageing was about 250 each year in the areas "closed" to
fishing and 500 in the "open" area. There may be improvement in the model with larger sample sizes. Different sample sizes can be generated from the survey ageing data by randomly replicating data values. The sensitivity of the model to increased sample sizes could be investigated.

### 7.4 Normal distribution of residuals.

The outliers in the quantile-quantile plot for red emperor, rankin cod, and lesser spangled emperor were due to zero catches in the first survey and significant catches in the second survey. Investigation of this data revealed no recording or analysis errors and there was no justification for them to be removed from the data set. The variation between catches in the same grid between the two surveys was probably due to fish movement within or between grids. In the present model, diffusion was allowed only between adjacent grids immediately after the first survey. It is possible to allow diffusion as well as mortality to occur several times for the one year period between the survey periods. This would allow greater diffusion and possibly improve the performance of the model.

### 7.5 Natural mortality (with efficiency) and spatial pattern of residuals

The values of $\mathrm{M}^{*}$ for red emperor were very low compared to the other species, due to the increased survey catch in 1994 over the whole survey area. The spatial plot of the residuals, revealed several high residual values in the east of the experimental area (grids $4633,4732,4734,4832$ ) which indicated the model was under-predicting in these eastern areas. An increase in efficiency over the whole survey area would result in low values of $\mathrm{M}^{*}$ but this would not explain the under-prediction of $q$ in the east. An efficiency increase together with migration of this species into the eastern, or a lower natural mortality in this eastern area, would be consistent with the parameter estimates. If this migration was from the central "open" area into the eastern closed area, the model would be over-estimating catchability, and hence fishing mortality.

The values of $M^{*}$ were low for lesser spangled emperor, suggesting an increase in efficiency in the second survey or migration of fish into the whole survey area. The residual values were low in the west for this species (the model over-estimating the predicted value in the west). This is consistent with a lower efficiency increase in the
west (where there are high concentrations of this species), a migration out of the west, or a higher natural mortality in this area.

The values of $M^{*}$ were very high for rosy threadfin bream. This species is concentrated in the far west of the fishery especially in shallow water landward of the fishery. The high values of $\mathrm{M}^{*}$ for this species are consistent with a large scale migration out of the fishery.

Future studies in this fishery include a tagging study, to validate ageing and to determine migration patterns. Changes in the age structure over time will also be investigated. This information will enable refinement of the model.

### 7.6 Effort distribution in 1994 and 1995

The level of effort between November 1993 and November 1994 was 18744 hours with $98 \%$ occurring in the fishing area from the pipeline to $117^{\circ} 30^{\prime}$ E. In 1995 the effort was concentrated in the west of the fishery due to the high catch rates and the proximity to the home port at Point Samson. The 1995 effort in the survey area (areas 1, 2, and 3 in Figure 2) was 18266 hours with a further 3443 hours fishing east of the experimental area (area 4 and 5 in Figure 2).


Figure 10. Fishing areas in the Pilbara Trawl Fishery

The fleet is expected to increase effort above the 1995 effort level and this will probably occur quite rapidly as the fleet is currently up-grading to larger vessels. The three most efficient boats in the fishery are catching at a rate of $135 \mathrm{~kg} / \mathrm{hour}$ and trawling 308 hours per month. If the whole fleet operated at this level for the full 84 boat months available in the fishery, the effort would be 25900 hours and the resulting catch would be 3500 tonne. If the distribution of this potential effort was the same as that in 1995, the effort levels in each of the five fishing areas would be that shown in Table 7.

Table 7. Effort distribution in 1994 and 1995 and the potential effort if the fleet operated like the most efficient boats.

Effort 1994 Effort 1995 Potential effort

| Area 1 | 7495 | 8644 | 10313 |
| :---: | :---: | :---: | :---: |
| Area 2 | 9311 | 4350 | 5190 |
| Area 3 | 1790 | 5272 | 6290 |
| Area 4 | - | 3113 | 3714 |
| Area 5 | - | 330 | 394 |
| Total | 18596 | 21709 | 25900 |

### 7.7 Management options

In order to formulate a management strategy, a number of options for redistribution and reduction of effort and their implication for the key species, is required.

Option 1. Allow effort to expand to full potential of access months with uncontrolled distribution of fishing. It is assumed that all vessels would achieve hours equivalent to the most effective current operators, the fleet would continue the pattern of access seen in 1995 where the fleet continues to concentrate effort on the traditional grounds off Point Samson. The expected impact of this fishing option can be assessed by comparing the fishing mortality (Table 8) with the values of $\mathrm{F}_{0.1}$ (Table 9). The number of hours trawling for this option is shown in Table 10. A rapid depletion of the large species would be expected. The small species, especially lesser spangled emperor would need monitoring for indications of decline.

Option 2. Allow effort to expand to full potential but redistribute effort to minimise impact on the most vulnerable species This would require reducing effort in the areas of largest concentration of the large species. The 1993 survey catch rates (kg/hour) of the five key species are shown in Table 11. The catch rates of red emperor are highest in Area 3 and second highest in area 2. For rankin cod, the catch rates are highest in Area 2.

Table 8. Largest value of average fishing mortality for 1995, for reduced effort levels, and biological reference point $\mathrm{F}_{0.1}$.

| Species | Option 1 | Option 2 | Option 3 | Option 4 | Option 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Flagfish | 0.121 | 0.091 | 0.077 | 0.073 | 0.074 |
| I s emperor | 0.253 | 0.213 | 0.195 | 0.193 | 0.195 |
| threadfin bream | 0.111 | 0.082 | 0.070 | 0.066 | 0.067 |
| red emperor | 0.194 | 0.140 | 0.116 | 0.109 | 0.110 |
| rankin cod | 0.330 | 0.242 | 0.203 | 0.191 | 0.194 |

Table 9. The minimum, median, and maximum values of the biological reference point $\mathrm{F}_{0.1}$.

|  | $\mathrm{F}_{0.1}$ |
| :---: | :---: |
| flagfish | $0.46,0.53,0.60$ |
| lesser spangled emperor | $0.45,0.50,0.55$ |
| threadfin bream | $0.52,0.60,0.70$ |
| red emperor | $0.10,0.12,0.15$ |
| rankin cod | $0.20,0.24,0.28$ |

Reduction of effort would need to be greatest in Area 2 and 3 to protect these large species and a reduction of $12 \%$, $29 \%$, and $39 \%$ in Areas 1, 2 , and 3 respectively is an illustrative example.

The 1995 logbook data indicated that catch rates of red emperor and rankin cod in Area 4 and 5 are similar to that in Area 3. Equivalent effort in these two areas can be determined by multiplying up on an area basis the effort in Area 3 ( multiply by 1.39 for area 4 and by 0.89 in area 5).

The values of average fishing mortality for this option (Table 8) are above the median values of $\mathrm{F}_{0.1}$ for the red emperor and rankin cod (Table 9). The expected impact would be slightly slowed depletion of the large species.

Option 3. An effort reduction of $16.7 \%$ of access time and redistribution of effort across the fishery. This could be achieved by an effort reduction in Areas 1, 2, and 3 of $16 \%, 51 \%$, and $52 \%$ respectively, with increased effort in Area 4 and 5 based on multiplying up the effort in Area 3. The values of average fishing mortality for this option (Table 8) are above the conservative (lower) values of $F_{0.1}$ for the large species (Table 9). This option would cause a decline in the large species to a smaller average size and tonnage.

Option 4. An effort reduction of $25 \%$ of access time and redistribution of effort across the fishery. This could be achieved by an effort reduction in Areas 1, 2, and 3 of $16 \%, 53 \%$, and $60 \%$ respectively, with increased effort in Area 4 and 5 based on multiplying up the effort in Area 3. The values of average fishing mortality for this option (Table 8) are equal to the conservative (lower) values of $\mathrm{F}_{0.1}$ for the large species (Table 9). This option would possibly secure the large species catch at existing levels. There would be a reduced catch of flagfish, with possible underexploitation. Lesser spangled emperor would have little reduction in catch due to the concentration of this species in Area 1. Threadfin bream would probably be underexploited.

Option 5. Buy out of one 6-month licence and then a $17 \%$ effort reduction of the remaining fleet and similar distribution of effort to Option 4. This would have a similar impact on the stocks as Option 4 (Table 8).

## Deepwater slope

The species in the deeper water (seaward of the 100 m depth contour) are expected to be highly vulnerable to over-fishing. It was recommended to industry that there be a moratorium on fishing in this area (Area 6 in Figure 10) until the extent and nature of this resource is determined.

Table 10. Effort (hours) in the 5 fishing areas for five different management options Option 1 Option 2 Option 3 Option 4 Option 5

| Area 1 | 10313 | 9075 | 8663 | 8456 | 8560 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## 35

| Area 2 | 5190 | 3882 | 2543 | 2335 | 2387 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Area 3 | 6290 | 3764 | 3019 | 2516 | 2642 |
| Area 4 | 2700 | 3764. | 3019 | 2516 | 2642 |
| Area 5 | 1401 | 5415 | 4340 | 3617 | 3797 |
| Total | 25900 | 25900 | 21584 | 19440 | 20028 |

The number of hours trawl effort for the five management options are shown Table 10. When a management plan is instigated at the beginning of 1997, the trawl effort in each area will need to be monitored by the owners of the licences and also by the Fisheries Department. A maximum total access time in the fishery, as well as a percentage access in each area was suggested to industry.

Table 11. Catch rates (kg/hour) in the survey area for 1993 survey.

|  | Area 1 | Area 2 | Area 3 |
| :---: | :---: | :---: | :---: |
| flagfish | 6.8 | 13.1 | 7.7 |
| lesser spangled emperor | 38.5 | 21.4 | 11.2 |
| threadfin bream | 17.9 | 22.5 | 13.1 |
| red emperor | 5.9 | 8.8 | 9.9 |
| rankin cod | 1.0 | 2.5 | 1.7 |

### 8.0 CONCLUSION

The unique experimental approach used in this project successfully provided initial estimates of the fishing mortality of key species in this multispecies fishery. It was determined that even with expansion of the fleet into hitherto unfished areas within the boundaries of the fishery, red emperor and rankin cod would be over-exploited, however there was scope for flagfish and particularly rosy threadfin bream to provide greater production. Lesser spangled emperor needs careful monitoring as its concentration in the west of the fishery makes it more susceptible to over-fishing than the other small species.

There is considerable scope for refinement of the fishing mortality estimates by investigation of the effect on the mortality estimates of using ages based on otolith weight and by exploring ways of calculating the fishing mortality estimates simultaneously for all age classes.

The results of the experiment provided a sound basis for the establishment of remedial management strategies, however considerable uncertainty still remains. To reduce this uncertainty, further research is required to assess the ongoing response of the stocks to the management strategies. The assumption made for this project, that bands on the otoliths are annual, needs to be validated. The degree to which the spatial distribution of the key species was affected by migration is unknown. Further assessment of the catch rates from the fishery and the tagging of the key species would provide the opportunity for obtaining additional information on movement patterns and refinement of values of fishing mortality

Yield-per-recruit models were used to interpret the impact on the fishery of different levels of effort. These models are restricted to the use of age and growth data. With additional biological data and the gathering of ongoing age composition data, further development and refinement of these models will be possible.

The effective management of the fishery will require assessment of the responses of the stocks to management options. It will be possible to make these assessments with more certainty in the future with additional information on the species enabling refinement and development of stock assessment models.

### 9.0 BENEFITS

The beneficiaries of the project are the commercial fishers on the NW Shelf. The benefit being the sustainability of their fishery

### 10.0 INTELLECTUAL PROPERTY

No commercially-saleable intellectual property is expected from the project.

### 11.0 STAFF EMPLOYED

| Mr Peter Stephenson | Mr lain Dunk |
| :--- | :--- |
| Mr Neil Sumner | Mr Norm Hall |
| Mr Tony Paust | Mr Ken Bryers |
| Mr Justin Chidlow | Mr Daryn Payne |

### 12.0 FINANCIAL STATEMENT

Total expenditure for this project from the Fisheries Research and Development Fund was $\$ 504,126.12$ including $\$ 181,029.05$ for 1993-94 from the WA FRDC Trust Account.

### 13.0 DISTRIBUTION LIST

AIMS, Townsville
Australian Fish Management Authority, Canberra
Division of Fisheries, CSIRO, Hobart
Department of Primary Industry and Fisheries, Darwin
Karratha Community Library
Nickol Bay Professional Fishermen's Association
QDPI, Cairns

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16.0 Appendix 1 Effort (hours) from the trawl fleet between November 1993 and November 1994 together with catch rates of the key species (kg/hour) from 1993 survey

| grid | effort | flagfish | lesser spangled emperor | rosy threadfin bream | red emperor | $\begin{aligned} & \text { rankir } \\ & \text { cod } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4432 | 10 | 3 | 4 | 0 | 2 | 0 |
| 4433 | 14 | 15 | 5 | 0 | 9 | 1 |
| 4434 | 19 | 15 | 18 | 6 | 5 | 2 |
| 4527 | 277 | 15 | 1 | 2 | 4 | 0 |
| 4528 | 385 | 7 | 0 | 0 | 4 | 1 |
| 4529 | 42 | 4 | 0 | 0 | 2 | 0 |
| 4530 | 44 | 1 | 0 | 0 | 2 | 0 |
| 4531 | 185 | 6 | 4 | 1 | 3 | 0 |
| 4532 | 22 | 9 | 5 | 3 | 4 | 1 |
| 4533 | 18 | 20 | 25 | 30 | 14 | 2 |
| 4534 | 23 | 22 | 16 | 27 | 11 | 3 |
| 4624 | 244 | 2 | 0 | 0 | 5 | 0 |
| 4625 | 849 | 6 | 3 | 1 | 1 | 1 |
| 4626 | 926 | 5 | 30 | 6 | 9 | 1 |
| 4627 | 195 | 3 | 4 | 5 | 2 | 1 |
| 4628 | 679 | 11 | 11 | 3 | 7 | 0 |
| 4629 | 638 | 10 | 22 | 4 | 5 | 0 |
| 4630 | 433 | 7 | 10 | 5 | 4 | 0 |
| 4631 | 538 | 10 | 16 | 17 | 10 | 1 |
| 4632 | 18 | 13 | 7 | 21 | 15 | 1 |
| 4633 | 20 | 10 | 11 | 38 | 17 | 2 |
| 4634 | 22 | 22 | 9 | 37 | 16 | 3 |
| 4721 | 4 | 4 | 0 | 0 | 21 | 0 |
| 4722 | 9 | 10 | 37 | 4 | 7 | 1 |
| 4723 | 33 | 12 | 39 | 4 | 4 | 2 |
| 4724 | 600 | 4 | 11 | 2 | 2 | 0 |
| 4725 | 847 | 6 | 44 | 14 | 5 | 0 |
| 4726 | 427 | 6 | 39 | 40 | 11 | 1 |
| 4727 | 539 | 10 | 44 | 8 | 19 | 1 |
| 4728 | 665 | 19 | 22 | 16 | 9 | 2 |
| 4729 | 631 | 21 | 49 | 31 | 9 | 4 |
| 4730 | 491 | 11 | 20 | 16 | 14 | 4 |
| 4731 | 365 | 10 | 15 | 19 | 20 | 4 |
| 4732 | 18 | 16 | 10 | 39 | 23 | 5 |
| 4733 | 21 | 8 | 5 | 27 | 10 | 6 |
| 4734 | 11 | 16 | 8 | 23 | 14 | 7 |
| 4821 | 20 | 12 | 51 | 2 | 9 | 1 |
| 4822 | 20 | 13 | 47 | 4 | 6 | 2 |
| 4823 | 44 | 22 | 156 | 11 | 11 | 2 |
| 4824 | 68 | 23 | 173 | 26 | 7 | 4 |
| 4825 | 1156 | 7 | 49 | 29 | 3 | 2 |
| 4826 | 1279 | 9 | 75 | 32 | 9 | 1 |
| 4827 | 1188 | 15 | 22 | 33 | 8 | 3 |


| 4828 | 1170 | 18 | 25 | 49 | 13 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4829 | 729 | 14 | 17 | 35 | 8 | 7 |
| 4830 | 472 | 7 | 13 | 23 | 7 | 6 |
| 4831 | 480 | 1 | 6 | 5 | 4 | 1 |
| 4832 | 8 | 0 | 0 | 5 | 2 | 0 |
| 4920 | 15 | 10 | 41 | 42 | 7 | 1 |
| 4921 | 18 | 10 | 121 | 16 | 7 | 1 |
| 4922 | 19 | 24 | 155 | 33 | 9 | 4 |
| 4923 | 18 | 19 | 139 | 33 | 5 | 1 |
| 4924 | 20 | 2 | 59 | 48 | 1 | 1 |
| 4925 | 89 | 7 | 29 | 24 | 5 | 3 |
| 4926 | 895 | 8 | 21 | 19 | 8 | 1 |
| 4927 | 334 | 17 | 33 | 22 | 9 | 2 |
| 4928 | 332 | 7 | 17 | 52 | 8 | 2 |
| 4929 | 66 | 9 | 45 | 28 | 10 | 24 |
| 5020 | 14 | 11 | 52 | 38 | 5 | 4 |
| 5021 | 14 | 10 | 112 | 23 | 10 | 0 |
| 5022 | 14 | 39 | 211 | 48 | 5 | 1 |

### 17.0 Appendix 2

## Weighted average effort.

The logbook effort of the fishing fleet in each trawl grid $\mathrm{E}_{\mathrm{i}}$ between November 1993 and November 1994 was weighted using the catch rate from the 1993 survey, $u_{i, k}$, in each grid $\mathrm{i}=1 \ldots \mathrm{n}$ for each species $\mathrm{k}=1 \ldots 5$. The weight, $\mathrm{w}_{\mathrm{i}, \mathrm{k}}$, is given by

$$
\begin{equation*}
\mathrm{w}_{\mathrm{i}, \mathrm{k}}=\mathrm{n} \cdot \frac{\mathrm{u}_{\mathrm{i}, \mathrm{k}}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}, \mathrm{k}}} \tag{5}
\end{equation*}
$$

The variable n is included in the weight so that the sum of the weights is equal to n .

The average effort for each species over the whole survey area, $\bar{E}_{k}$, is given by

$$
\begin{equation*}
\bar{E}_{k}=\frac{\sum_{i=1}^{n} w_{i, k} E_{i}}{n} \tag{6}
\end{equation*}
$$

and the variance of the average effort for each species, $V\left(\bar{E}_{k}\right)$, is given by

$$
\begin{equation*}
V\left(E_{k}\right)=\frac{1}{n-1} \sum_{i=1}^{n} w_{i, k}\left(E_{i}-\bar{E}_{k}\right)^{2} \tag{7}
\end{equation*}
$$

## Fishing mortality

The fishing mortality, $\mathrm{F}_{\mathrm{k}, \mathrm{t}}$, for species k and age class t , is the product of the average fishing effort for a particular species, $\bar{E}_{k}$ (in hours), and the catchability, $\mathrm{q}_{\mathrm{k}, 1}$, for each species $k$ and age class $t$ :

$$
\begin{equation*}
F_{k, t}=\left(\bar{E}_{k}\right)\left(q_{k, t}\right) \tag{8}
\end{equation*}
$$

The sample variance for the average fishing mortality, $V\left(F_{k, t}\right)$, is obtained using the formula in Kendall and Stuart (1969) and is given by:

$$
\begin{equation*}
V\left(F_{k, t}\right)=F_{k, t}{ }^{2}\left(\frac{V\left(\bar{E}_{k}\right)}{\bar{E}_{k}{ }^{2}}+\frac{V\left(q_{k, t}\right)}{q_{k, t}{ }^{2}}\right) \tag{9}
\end{equation*}
$$

where $q_{k, t}$ and $V\left(q_{k, t}\right)$ are the catchability and the variance of the catchability obtained from the model. To obtain the confidence limits of $\mathrm{F}_{\mathrm{k}, \mathrm{t}}$, the degrees of
freedom of the smallest sample was used, in this case the degrees of freedom of $q_{k, t}$. The $100(1-\alpha) \%$ confidence limits for $F_{k, t}$ are given by

$$
\begin{equation*}
F_{k, t} \pm\left(V\left(F_{k, t}\right)\right)^{1 / 2} t_{n-p, \alpha / 2} \tag{10}
\end{equation*}
$$

where $t_{n-p, \alpha / 2}$ denotes the upper $\alpha / 2$ percentile of the $t$ distribution with $n-1$ degrees of freedom.

The fishing mortality could be estimated for only a restricted number of age classes by the model. To obtain estimates of the fishing mortality over the whole age range, a logistic function was jointly fitted to the fishing mortalities from the model and to the observed sample age distribution. The age distribution of the species was assumed to result from a population with constant recruitment and constant natural mortality exposed to a sequence of fishing mortalities proportional to the observed fishing effort.

The logistic function is given by

$$
\begin{equation*}
\hat{F}_{t}=\frac{a}{1+b e^{-c t}} \tag{11}
\end{equation*}
$$

where $\hat{F}_{t}$ is the estimate of fishing mortality at age $t$.

For a fixed natural mortality and initial guesses of fishing mortality for all age classes in the age-frequency distribution, the relative number of fish of each age expected in the catch, $\hat{\mathrm{C}}_{\mathrm{t}}$, was determined from

$$
\begin{equation*}
\hat{\mathrm{C}}_{\mathrm{t}}=\hat{F}_{\mathrm{t}} \mathrm{~N}_{\mathrm{t}} \tag{12}
\end{equation*}
$$

where $N_{t}$ is the number of fish of age $t$, and

$$
\begin{equation*}
N_{t}=N_{t-1} \exp \left(-Z_{t-1}\right), t>0 \tag{13}
\end{equation*}
$$

where $Z_{t}=M+F_{t}$ is total mortality and $N_{0}$ is set to the value 1 .

The proportion of fish in each age class, $\hat{p}_{t}$, was determined by

$$
\begin{equation*}
\hat{\mathrm{p}}_{\mathrm{t}}=\frac{\hat{\mathrm{C}}_{\mathrm{t}}}{\sum_{\mathrm{t}=0}^{\mathrm{t}_{a}} \hat{\mathrm{C}}_{\mathrm{t}}} \tag{14}
\end{equation*}
$$

For each species, an estimate of the probability of the observed sample from a multinominal probability distribution (Mendenhall et al. 1986) was calculated as

$$
\begin{equation*}
L 1(a, b, c)=\frac{\left(n_{1}+n_{2}+n_{3}+\cdots n_{\alpha}\right)!}{n_{1}!n_{2}!n_{3}!\cdots n_{\alpha}!} \hat{p}_{1}^{n_{1}} \hat{p}_{2}^{n_{2}} \hat{p}_{3}^{n_{3}} \cdots \hat{p}_{\alpha}^{n_{\alpha}} \tag{15}
\end{equation*}
$$

where $n_{1}, n_{2}, n_{3}, \cdots n_{\alpha}$ are the frequencies of the combined 1993 and 1994 survey age-frequency distributions for age classes 1 to $\alpha$.

For the age classes where fishing mortalities, $F_{t}$, were obtained from the model, the likelihood function (Freund 1992) given by

$$
\begin{equation*}
L 2(a, b, c)=\prod_{t=\beta}^{\delta}\left(\frac{1}{\sqrt{2 \pi \sigma_{t}^{2}}} \exp \left[\frac{-\left(\hat{F}_{t}-F_{t}\right)^{2}}{2 \sigma_{t}^{2}}\right]\right) \tag{16}
\end{equation*}
$$

was determined, where $F_{t}$ and $\sigma_{t}{ }^{2}$ are the parameter estimate of the fishing mortality and its variance for the age class $t$ determined from the model, and $\beta, \delta$ are the minimum and maximum age classes for which fishing mortalities were determined from the model. The variance $\sigma_{t}{ }^{2}$ is assumed to approximate the residual variance of the estimate of $F_{t}$ from the value calculated from the logistic equation.

For ease of calculation, the natural logarithms were used, the functions used being

$$
\begin{array}{r}
\ln (\mathrm{L} 1)=\ln \left(\left(\mathrm{n}_{1}+\mathrm{n}_{2}+\cdots \mathrm{n}_{\alpha}\right)!\right)-\ln \left(\mathrm{n}_{1}!\mathrm{n}_{2}!\cdots \mathrm{n}_{\alpha}!\right)+\mathrm{n}_{1} \ln \left(\hat{\mathrm{p}}_{1}\right)+\mathrm{n}_{2} \ln \left(\hat{\mathrm{p}}_{2}\right)+\cdots+\mathrm{n}_{\alpha} \ln \left(\hat{\mathrm{p}}_{\alpha}\right) \\
\ln (\mathrm{L} 2)=-\sum_{\mathrm{t}=\beta}^{\delta}\left(\ln \left(\sqrt{2 \pi \sigma_{\mathrm{t}}^{2}}\right)+\frac{\left(\hat{\mathrm{F}}_{\mathrm{t}}-F_{\mathrm{t}}\right)^{2}}{2 \sigma_{\mathrm{t}}^{2}}\right) \tag{18}
\end{array}
$$

The estimates of the parameters, $a, b$, and $c$ of the logistic function (equation 11) were found by maximisation of the product of the probability functions $\ln (L 1)$ and $\ln (\mathrm{L} 2)$.

